Hierarchy of graphs by spatial embeddings

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	outer-planar	planar	1-apex	2-apex
disk condition	S^1-flat $\rightarrow$	outer-flat $\rightarrow$	flat $\rightarrow$	primitive
fundamental group	totally free	totally free	totally free	connected free
subgraph condition	S^1-linkless $\rightarrow$	outer-linkless $\rightarrow$	linkless $\rightarrow$	knotless
obstruction set	K_4 $K_{3,2}$	K_5 $K_{3,3}$	K_6-family $K_{3,3,1}$-family	K_7-family $K_{3,3,1,1}$-family Foisy, etc.
Colin de Verdiere	$\mu(G) \leqq 2$	$\mu(G) \leqq 3$	$\mu(G) \leqq 4$	$\mu(G) \leqq 5$?
move	1-switch	2-switch	3-switch	4-switch?
unique embedding	2-connected	3-connected	4-connected	5-connected?

Petersen family

$Y\Delta$- and ΔY-exchange

$$K_7 \rightarrow H_8 \rightarrow H_9 \rightarrow H_{10} \rightarrow H_{11} \rightarrow \boxed{H_{12}}$$

$$F_9 \rightarrow F_{10}$$

$$E_{10} \rightarrow E_{11}$$

$$C_{11} \rightarrow C_{12} \rightarrow C_{13} \rightarrow \boxed{C_{14}}$$

K_7-family

$K_{3,3,1,1}$-family
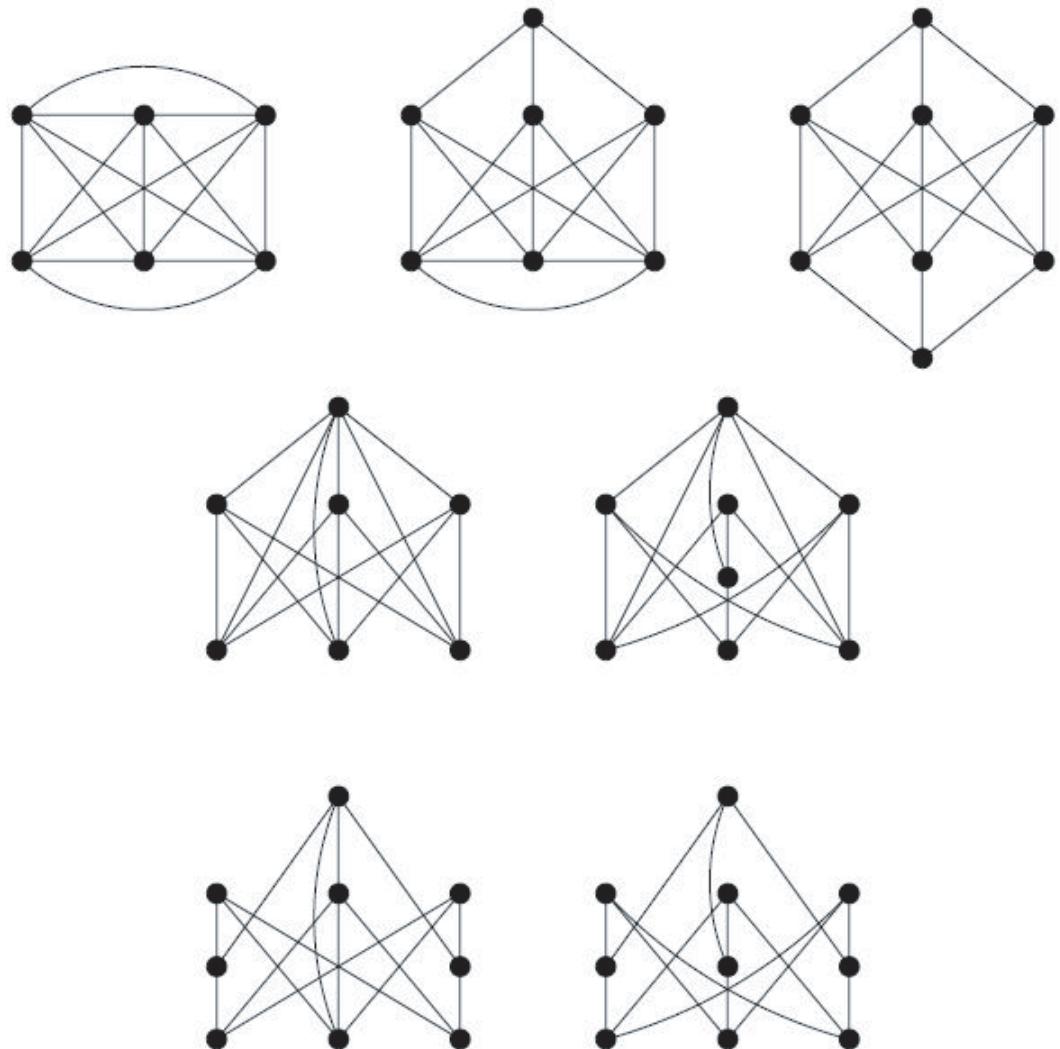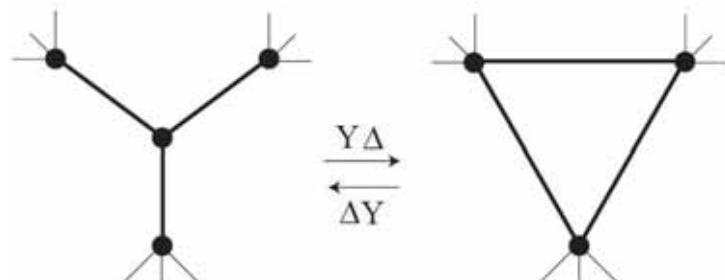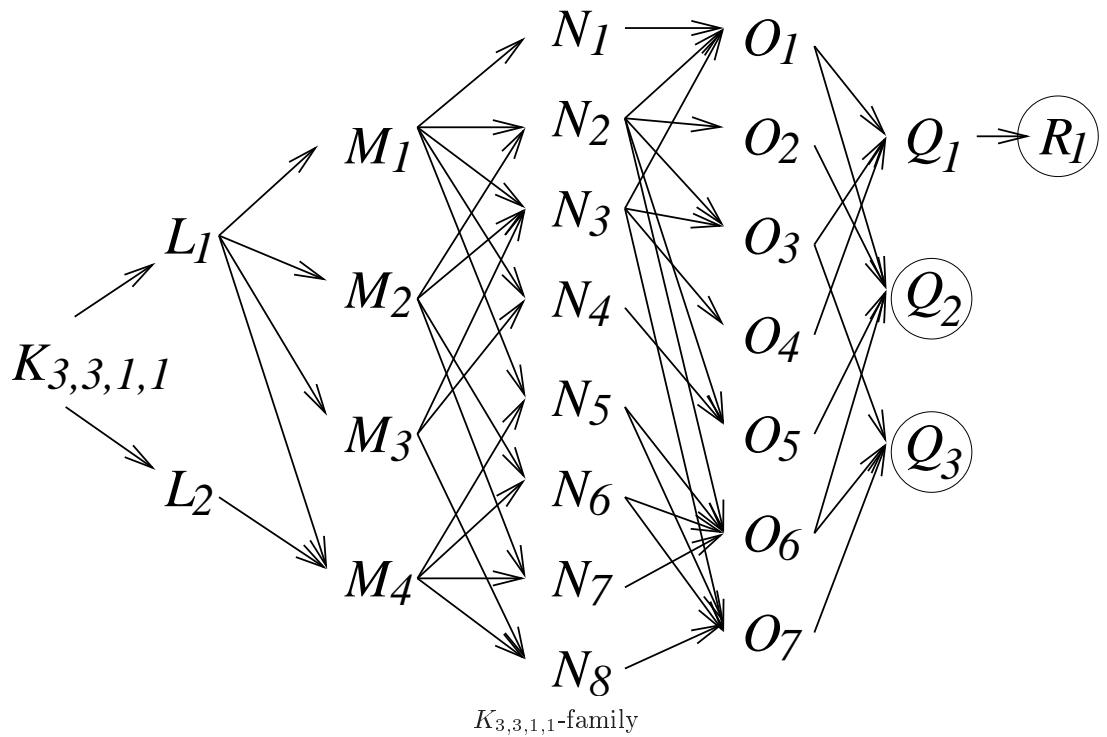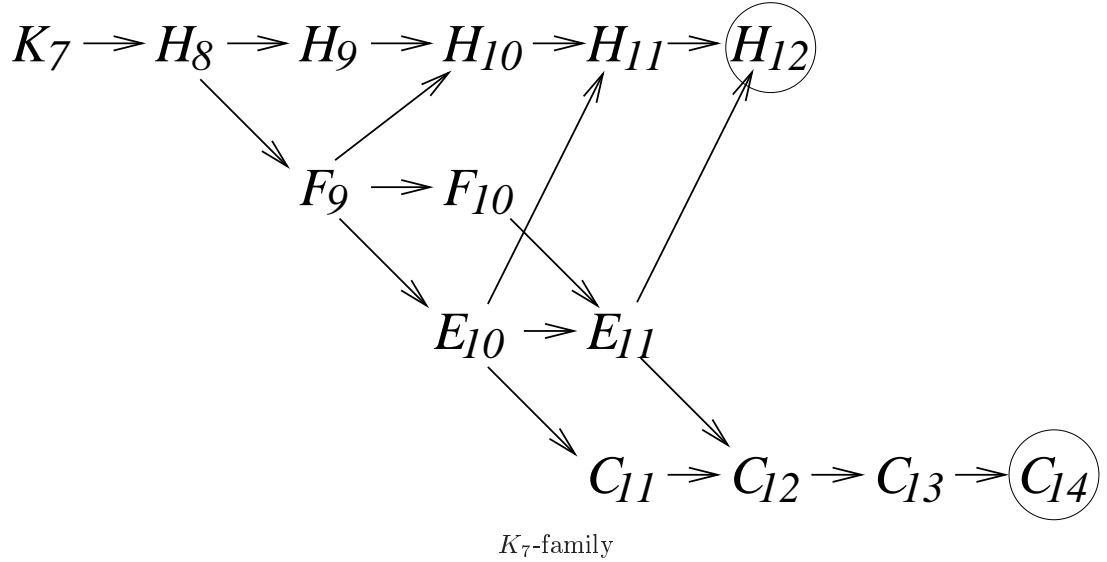
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